LAST NAME SOLUTION Alphabetic # ____________

FIRST NAME ________________________ UIN ________________________

Midterm Exam 2 – April 2018

1 NOTE SHEET AND CALCULATOR ALLOWED

CLOSED BOOK, CLOSED NOTES

NO CELL PHONES OR OTHER ELECTRONIC DEVICES PERMITTED

1.________/ 25

2.________/ 25

3.________/ 25

4.________/ 25

Total ________/ 100

Bonus.________/

Notes:

1. Hard-hitting, direct bullet thoughts are preferred when explanations are called for.
2. Note: Potentially useful constants and tables are included on the last page

NAME *SOLUTION*

1. (25 pts total) Engineering Economics.

   a. An eight-year project will requires an initial $10,000 investment. Annual expected revenue is expected to be $3,000.

      1) (5 pts) What is the net present value of the project assuming a 7 % discount rate?

USE $(P|A, d\%, n)$: $P = A \cdot \frac{(1+d)^n - 1}{d(1+d)^n} = 3000 \cdot \frac{(1.07)^8 - 1}{(.07)(1.07)^8} = 3000 \cdot \frac{(1.718 - 1)}{.07(1.718)}$

$P = 3000 \cdot 5.9713 = 17{,}913$

NPV = 17913 - 10000 = $7,913 ANS.

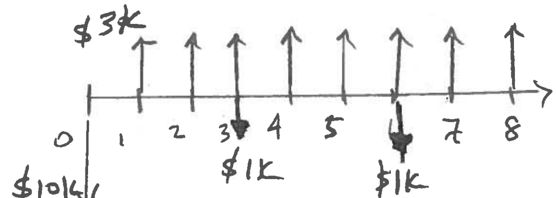
      2) (5 pts) How much will the net present value of a project decrease if a maintenance cost of $1000 per year is incurred at the end of year 3 and year 6? Assume that the discount rate is 7%.

$PV_{MAINT\ COST} = \frac{1000}{(1.07)^3} + \frac{1000}{(1.07)^6} = 1000\left(\frac{1}{1.225} + \frac{1}{1.500}\right) = 1000(1.482)$

= $1482 ANS. ← DECREASE IN NPV i.e. NPV = 7913 - 1482 = $6,430

      3) (5 pts) Draw the cash flow diagram including elements of 1) and 2) above.

$10K INITIAL INVESTMENT.
$3K ANNUAL PAYMENTS
$1K MAINT COST @ YEARS 3 & 6



   b. (5 pts) An efficient refrigerator has an initial cost of $1240, but will provide $200 of savings annually. Assuming a lifetime of 8 years, what is the internal rate of return (approximately)?

USE THE APPROACH THAT $P = \$\text{INITIAL COST} + \$\text{SAVED}(P/A, i, n) = 0$

$(P/A, i, n) = \frac{\$\text{INITIAL COST}}{\$\text{SAVED}}$

$= \frac{1240}{200} = 6.2$

USE IRR TABLE, $n = 8$

$\Rightarrow$ IRR ≈ 6%

MISSING 5 PTS??
OR ADJUST POINTS

NAME SOLUTION

2. (25 pts total) You are considering installing two types of turbines:
   - 100 kW rated output, 50 m hub height, D = 20 m
   - 200 kW rated output, 80 m hub height, D = 30 m.

Suppose the wind is blowing at 9.42 m/s at 10 m on terrain covered by tall grass(α=0.15). Assume both turbines operate with an efficiency of 29% and standard air density, 1.225 kg/m^3 .

a. (6 pts) Estimate the wind velocity at each turbine's hub height.

EQN 7.18 $\left(\frac{v}{v_0}\right)=\left(\frac{H}{H_0}\right)^{\alpha}$

$v_0 = 9.42$ m/s

$H_0 = 10$ m

(1) $v_1 = v_0\left(\frac{50}{10}\right)^{.15} = 9.42(5)^{.15} = 11.99$ m/s ~ 12 m/s ANS.

(2) $v_2 = v_0\left(\frac{80}{10}\right)^{.15} = 9.42(8)^{.15} = 12.87$ m/s ANS.

b. (6 pts) Using your answer from (a), calculate each turbine's output power, if they are not shedding wind.

EQN 7.27 $P_b = C_p \frac{1}{2}\rho A v^3$

$P_1 = C_p\frac{1}{2}\rho A_1 v_1^3 = .29\left(\frac{1}{2}\right)1.225(100\pi)(11.99)^3$

$= 96.1$ kW ANS.

$A_1 = \pi R^2 = \pi\left(\frac{D}{2}\right)^2 = \pi\left(\frac{20}{2}\right)^2 = 100\pi$

$P_2 = C_p\frac{1}{2}\rho A_2 v_2^3 = .29\left(\frac{1}{2}\right)1.225(225\pi)(12.87)^3$

$= 267.6$ kW ANS.

$A_2 = \pi\left(\frac{30}{2}\right)^2 = 225\pi$

c. (6 pts) Assume each turbine's average windspeed is ½ of the windspeed you found (a) and Rayleigh statistics. Estimate the average power output of each turbine.

EQN 7.49 $\bar{P}_{WIND} = \frac{6}{\pi}\left(\frac{1}{2}\rho A\bar{v}^3\right)$

$\bar{P}_1 = \frac{6}{\pi}\left(\frac{1}{2}\right)(C_p\rho)A_1\bar{v}_1^3 = \frac{6}{\pi}\left(\frac{1}{2}\right)1.225(100\pi)(6)^3 .29$

$= 23.01$ kW ANS.

$P_{OUT} = C_p\frac{6}{\pi}\frac{1}{2}\rho A\bar{v}^3$ (C_p ← TURBINE EFFICIENCY)

$P_2 = \frac{6}{\pi}\left(\frac{1}{2}\right)C_p\rho A_2\bar{v}_2^3 = \frac{6}{\pi}\left(\frac{1}{2}\right)(1.225)(225\pi)(6.435)^3 = 63.87$ kW ANS.

d. (7 pts) Using the rated powers and your answer in (c), estimate the capacity factor and each turbine's annual energy production.

EQN 7.63 $CF = .087\bar{v} - \frac{P_R(kW)}{D^2}$

$CF_1 = .087\bar{v}_1 - \frac{P_{R1}}{D_1^2} = .087(6) - \frac{100}{20^2}$

$= .272$ ANS

$CF_2 = .087 v_1 - \frac{P_{R2}}{D_2^2} = .087(6.435) - \frac{200}{(30)^2}$

$= .337$ ANS.

$E_1 = CF_1 \cdot P_{R1} \cdot 8760$

$= .272 \cdot 100 \cdot 8760$

$= 238.2$ MWhr/yr ANS.

$E_2 = CF_2 \cdot P_{R2} \cdot 8760$

$= .337 \cdot 200 \cdot 8760$

$= 590.4$ MWhr/yr

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3. (25 pts total) Wind Power Fundamentals

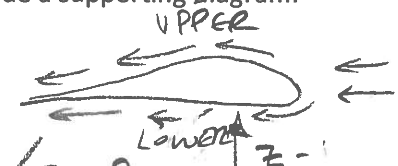
a. Using Bernoulli's principle:

$$\frac{1}{2}\rho v^2 + \rho g z + p = \text{constant}, \quad \textit{where}$$

$v \triangleq$ is the fluid flow speed at a point on a streamline,
$g \triangleq$ is the acceleration due to gravity,
$z \triangleq$ is the distance above a reference plane
$p \triangleq$ is the pressure at the chosen point,
$\rho \triangleq$ is the fluid density at all points in the fluid,

1) (5 pts) Develop a mathematical expression describing lift on a wing; include a supporting diagram.

THE AIR MOVING OVER WING'S UPPER SURFACE TRAVELS A GREATER DISTANCE PRODUCING LIFT.



$$\frac{1}{2}\rho v_u^2 + \rho g z + P_u = \text{CONSTANT} = \frac{1}{2}\rho v_\ell + \rho g z + P_\ell$$

DISTANCE $z_u \approx z_\ell$

$$\frac{1}{2}\rho v_u^2 + P_u = \frac{1}{2}\rho v_\ell^2 + P_\ell$$

$$\frac{1}{2}\rho(v_u^2 - v_\ell^2) + (P_u - P_\ell) = 0 \Rightarrow$$

IF $v_u > v_\ell$, THEN $P_u < P_\ell$ MUST BE TRUE.

2) (2 pts) List and briefly describe 2 of the 4 underlying assumptions inherent in applying Bernoulli's principle to explain lift.

- STEADY FLOW, i.e. AIR VELOCITY AT A POINT CANNOT CHANGE w/ TIME
- THE FLOW MUST BE INCOMPRESSIBLE – THOUGH THE PRESSURE VARIES THE DENSITY ρ REMAINS CONSTANT ALONG A STREAMLINE.
- FRICTION CAUSED BY VISCOUS FORCES ARE NEGLIGIBLE
- A FLUID ELEMENT MUST BE ALONG THE FLOW – i.e. NO TURBULENCE

3) (3 pts) What was Albert Betz's contribution to wind power conversion theory?

HE DEVELOPED AN UPPER BOUND ON THE MAXIMUM POWER THAT COULD BE EXTRACTED BY A TURBINE BLADE. (NO REAL SYSTEM ACHIEVES THIS)

$C_p = 59.3\%$

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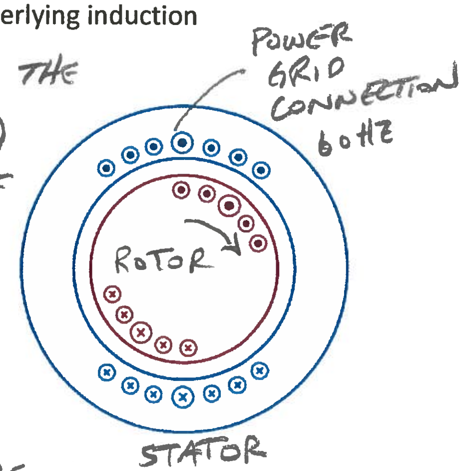
3. (cont)

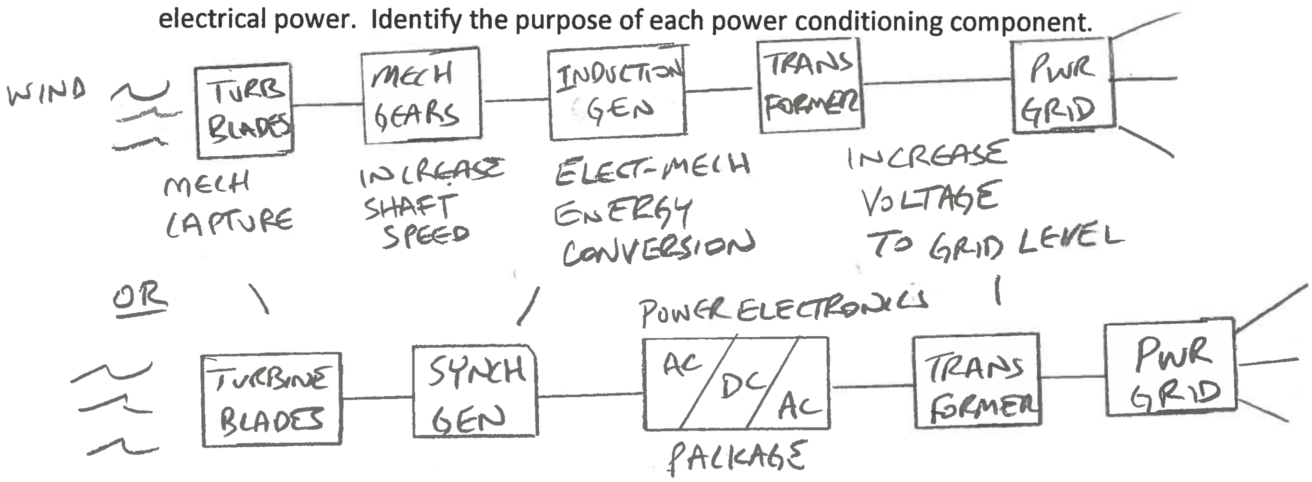
b. Electromechanical Energy Conversion

1) (5 pts) Describe how the **induction generator** (3 phase, 2-pole) meets the frequency conditions for non-zero average power (use a simple mathematical expression, complete the sketch/figure, and provide supporting words including the **electrical law / phenomenon** underlying induction generator function)



- STATOR WINDINGS ARE ELECTRICALLY CONNECTED TO THE POWER GRID; GRID OPERATES @ ω_s (60Hz)
- 3ϕ STATOR WINDINGS CONNECTED TO GRID CAUSE A MAGNETIC FIELD ROTATING AT ω_s
- PRIME MOVER TURNS ROTOR AT SPEED ω_m WHICH IS FASTER THAN ω_s
- BECAUSE ROTOR EXPERIENCES A CHANGING MAGNETIC FIELD ($\omega_m \neq \omega_s$), VOLTAGES ARE INDUCED IN ROTOR WINDING (FARADAY'S LAW)
- $\omega_m = \omega_s + \omega_r$ OR $\omega_r = \omega_m - \omega_s$

2) (5 pts) Draw a sequence of **power-conditioning components** of a nominal wind energy conversion system connected to the power grid depicting the stages needed to convert wind power to electrical power. Identify the purpose of each power conditioning component.



3) (5 pts) Can a **synchronous generator** used in an effective wind energy conversion system? Explain your reasoning.

YES. AN IEEE TYPE 4 WIND ENERGY CONVERSION SYSTEM USES A SYNCHRONOUS GENERATOR – SEE DIAGRAM IN b2) IN CONJUNCTION W/ A POWER ELECTRONICS MODULE CONVERTING FROM GEN. AC SIGNAL TO DC TO 60HZ SIGNAL FOR POWER GRID

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4. (25 pts total) A horizontal-axis wind turbine with a 100-m diameter rotor is 30% efficient in 6 m/s winds at sea level (1 atmosphere pressure) and 25°C.

a. (6 pts) How much power would the generator produce in those winds?

$A = \pi R^2 = \pi \left(\frac{D}{2}\right)^2 = \pi (50)^2 = 2500\pi$

$P_T = C_p \cdot \frac{1}{2} \rho A v^3$

$= .3 \left(\frac{1}{2}\right)(1.225)(2500\pi)\, 6^3$

$= 311.7 \text{ KW}$ ANS.

b. (6 pts) How much power would same generator produce when sited on a 2500-m mountaintop at 10°C with a 6 *m/s* wind, having corrected for the change in air density?

EQN 7.17: $\rho\left(\frac{kg}{m^3}\right) = \frac{353.1\, e^{-.0342 Z / T}}{T\,(°K)}$

$\rho_{2500} = \frac{353.1\, e^{-.0342 Z / T}}{T} = \frac{353.1\, e^{-.0342(2500)/(273+10)}}{273+10} = .922 \text{ kg/m}^3$

$P_{T,2500m} = .3\left(\frac{1}{2}\right)(.922)(2500\pi)\, 6^3 = 234.7 \text{ kW}$ ANS.

c. (6 pts) Assuming sea level siting, 7 *m/s* average wind speed, and Rayleigh wind statistics, what is the optimum generator size?

EQN 7.66: $P_R \text{(kW)} = .0435 \bar{V} D^2$

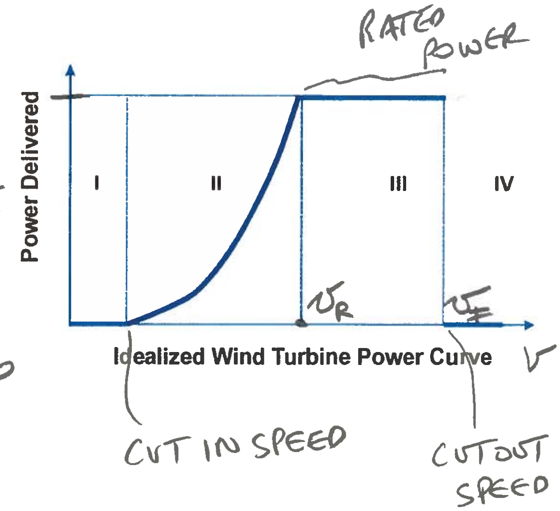
$P_R \text{(kW)} = .0435 \bar{V} D^2$

$= .0435\ 6\ (100)^2 = 3{,}045 \text{ KW}$ ANS OR 3.045 MW

d. (3 pts) Define capacity factor (CF).

$CF = \frac{\text{ENERGY DELIVERED}}{\text{ENERGY @ FULL POWER}}$

e. (4 pts) Explain why a CF of 1 cannot be achieved based upon the idealized wind turbine power curve.



THE WIND IS A VARIABLE RESOURCE THAT IS UNLIKELY TO BE AT A SPEED @ OR ABOVE THE SYSTEMS RATED SPEED CONTINUALLY THROUGH 8760 HOURS IN A YEAR.

(ie $V_{RATED} \le v \le V_{FURL}$ THROUGHOUT A FULL YEAR)

NAME ___________________________

Useful Constants, tables, and figures

One cubic foot	=	7.4805 gallons	0.02832 m3
One foot per second	=	0.6818 mph	0.3048 m/s
Water density	=	652.428 lb/ft3	1000 kg/m3
1 kW	=	737.56 ft-lb/s	1000 N-m/s
Gravity	=	32.2 ft/sec2	9.81 m/s2
One BTU	=	777.9 ft-lb	1055 J
1 PSI	=	2.307 feet of water.	
One Meter	=	39.37 inches	
One Kilogram	=	2.2 pounds	
One ton	=	2000 pounds	

Life (yr)	Internal Rate of Return (%) 3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	16%	18%	20%	22%
5	4.58	4.45	4.33	4.21	4.10	3.99	3.89	3.79	3.60	3.43	3.27	3.13	2.99	2.86
6	5.42	5.24	5.08	4.92	4.77	4.62	4.49	4.36	4.11	3.89	3.68	3.50	3.33	3.17
7	6.23	6.00	5.79	5.58	5.39	5.21	5.03	4.87	4.56	4.29	4.04	3.81	3.60	3.42
8	7.20	6.73	6.46	6.21	5.97	5.75	5.53	5.33	4.97	4.64	4.34	4.08	3.84	3.62
9	7.79	7.44	7.11	6.80	6.52	6.25	6.00	5.76	5.33	4.95	4.61	4.30	4.03	3.79
10	8.53	8.11	7.72	7.36	7.02	6.71	6.42	6.14	5.65	5.22	4.83	4.49	4.19	3.92
11	9.25	8.76	8.31	7.89	7.50	7.14	6.81	6.50	5.94	5.45	5.03	4.66	4.33	4.04
12	9.95	9.39	8.86	8.38	7.94	7.54	7.16	6.81	6.19	5.66	5.20	4.79	4.44	4.13
13	10.63	9.99	9.39	8.85	8.36	7.90	7.49	7.10	6.42	5.84	5.34	4.91	4.53	4.20
14	11.30	10.56	9.90	9.29	8.75	8.24	7.79	7.37	6.63	6.00	5.47	5.01	4.61	4.26
15	11.94	11.12	10.38	9.71	9.11	8.56	8.06	7.61	6.81	6.14	5.58	5.09	4.68	4.32
20	14.88	13.59	12.46	11.47	10.59	9.82	9.13	8.51	7.47	6.62	5.93	5.35	4.87	4.46
25	17.41	15.62	14.09	12.78	11.65	10.67	9.82	9.08	7.84	6.87	6.10	5.47	4.95	4.51
30	19.60	17.29	15.37	13.76	12.41	11.26	10.27	9.43	8.06	7.00	6.18	5.52	4.98	4.53